

Byram's Derivation of the Energy Criterion for Forest and Wildland Fires

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Abstract. G.M. Byram's energy criterion for forest and wildland fires consists of two equations: one for computing the rate of flow of kinetic energy in the atmosphere due to the wind field (P_w), and one for estimating the rate of conversion of thermal energy to kinetic energy in the convection column (P_f). The derivation of the equations has remained unpublished since their introduction in 1959. Byram considered the dimensionless ratio of P_f to P_w an indicator of the vertical structure of convection over a fire and referred to the ratio as the convection number, N_c . In view of past and current interest in the behavior of large fires, Byram's derivation of the equations for P_w and P_f is presented along with a sketch and some additional wording for purposes of clarification. The assumptions and possible limitations in Byram's analysis are discussed.

Keywords: Blowup fires; Buoyancy; Convection columns; Energy conversion

Introduction

G. M. Byram (1959) proposed equations for computing two rates of energy flow in a neutrally stable atmosphere at height z above a forest or wildland fire. One of these rates is the horizontal flow of kinetic energy through unit vertical area of the atmosphere at height z , denoted by P_w . The second energy flow is the rate of conversion of thermal energy to kinetic energy in the convection column, denoted by P_f . Byram referred to the ratio of these rates, P_f/P_w , as the energy criterion. According to Byram, case studies (unpublished) showed that when P_f/P_w equals or exceeds unity for at least 1,000 feet above a fire, the fire tends to exhibit erratic behavior or blow up. Though no derivation of the P_w and P_f equations was presented by Byram, their publication generated considerable interest in the United States (Chandler et al. 1963), Australia (Cheney and Bary 1969), and the Soviet Union (Frank

1964). Reifsnyder (1959), in a review of the Davis (1959) book on forest fire control and use, criticized Byram for publishing speculative and controversial material and for not providing a derivation of the equations or giving a reference where the derivation could be found. Steiner (1976), in a qualitative explanation of the relationship between blowup fires and atmospheric wind profiles, also noted that a derivation of the P_w and P_f equations had not been published, and further, that no connection had been made between the equations and previous research on fluid shear layers.

Byram (personal communication), in retrospect, simply regarded the results of his case studies so relevant to understanding and interpreting wildfire behavior that he felt they should appear in the chapter on forest fire behavior he was preparing for the Davis (1959) book on forest fire control. He considered the derivation of P_w and P_f which he referred to as complex and involving atmospheric physics and thermodynamics, inappropriate for the book. At the time, however, there existed a basis for the energy criterion in the work of Rouse (1947) on gravitational diffusion from a boundary source of heat. Furthermore, the analysis of Scesa (1957) had suggested a similar criterion for determining conditions for which buoyancy can be disregarded in the study of convection from a line source of heat at the earth's surface. Byram (1959) referred to Scesa's work near the end of his chapter on forest fire behavior.

Since its introduction in 1959, the energy criterion has appeared in several analyses of the behavior of large forest fires (Wade and Ward 1973, Simard et al. 1983, Aronovitch 1989). Forms of the criterion have been used to understand the processes of crowning and spotting (Kurbatsky 1969, Mateev and Mateev 1977, Albini 1983, Rothermel 1991a) and have appeared in analyses and models of surface fires and fluid flow downwind of surface line sources of heat (Thomas 1964, Telitsyn 1969, Albini 1981, Raupach 1990, Martin et al. 1991). The energy criterion also has been used to determine the burning conditions under which scaveng-

ing of particulate matter in the convection column by cumulus clouds can be expected (Radke and Ward 1991). The criterion was referred to by Byram et al. (1964) as the convection number, N_c , and will be so designated in the remainder of this paper. The convection number terminology was used by Wade and Ward (1973) and later by Aronovitch (1989), but has not been generally applied in forest fire research.

The ratio of the intensity of a line source of heat to the cube of the windspeed has been a relevant parameter in fields of study other than forest fire behavior. Rankine (1950) reported laboratory wind tunnel experiments in which windflow perpendicular to a line of burners was used to study the dispersal of fog over airport runways. In his analysis of the Rankine data, G.I. Taylor (1961) showed that the intensity/windspeed cubed ratio determined the angle of the upper edge of the buoyant downwind flow. Roberts (1979) also determined from experiments and from dimensional analysis that the reciprocal of the above ratio, which he referred to as a Froude number, governs the dispersion of sewage from ocean outfall diffusers. Any of 3 separate dispersion regimes can be observed, depending on the value of the Froude number.

It is probable that regimes of convection column appearance similar to the regimes of the Roberts (1979) study can be identified for forest and wildland fires. Research along these lines could improve our understanding and prediction of extreme wildfire behavior. One aspect of the required effort is the case study—wildfire observation approach used earlier by Byram (1959). A second part of the work is to determine the kinds of fires to which the present N_c concept may be applied and to develop more general methods of describing atmospheric-convective relationships above area and line sources of heat. Such studies have appeared only recently. For example, Aronovitch (1989) recommended that N_c be used to estimate convection column heights of wildfires. He developed a method of determining the profile of the convection column from values of P_w and P_f . Rothermel (1991a) developed a fire characteristics chart for wind-driven crown fires and presented evidence suggesting that the value of N_c can be used to distinguish between wind-driven and plume-dominated wildfires. In additional work, Rothermel (1991b) used weather and fire behavior data from several large wildfires and the Byram (1959) transition criterion $N_c = 1$ to separate wind-driven and plume-dominated fires in a manner that agreed with the observed convection column characteristics. He regarded a fire for which $N_c < 1$ as a wind-driven fire.

In view of past and present attention to N_c , it appears that publication of Byram's derivation would

be of interest to researchers concerned with the behavior of large wildland fires. Having had access to the derivation as a member of Byram's Southeastern Forest Experiment Station research project in the 1960's (George Byram retired in 1968), the author intends in this paper to present Byram's derivation of N_c with some wording added for purposes of clarification. A diagram also is provided to illustrate the main concepts of the derivation. It is clear that the derivation upon which this paper is based is not the original one because the reported P_w and P_f equations differ from those in Davis (1959). Furthermore, the version used here was prepared after 1964 because it uses N_c terminology which was first introduced in 1964 (Byram et al. 1964). The essence of the derivation is contained in the following four sections.

Energy Rate of the Wind Field P_w

Consider a fire heat source in the form of an infinitely long line in the y direction perpendicular to the flow of wind in the x direction. Such a fire and the associated column of smoke are shown in Figure 1. The wind field is given by $v(z)$ and the fire front is moving in direction x at a mean rate r . If there is no change in wind direction with height and the magnitude of the wind at height z is v , the kinetic energy per unit volume of air based on an air speed corrected for speed of the fire is

$$E_w = \rho[(v - r)^2]/2 \quad (1)$$

where ρ is mass density of the air at height z . Because unit volume of air travels distance dx and both the flame front and vertical plane $dydz$ ($dy = dz = 1$ in Figure 1) travel distance dx' in time interval dt , the horizontal flux of kinetic energy (or power of the wind) through the vertical plane $dydz$ corrected for speed r of the fire is given by \dot{E}_w (or P_w) as

$$\begin{aligned} P_w &= \dot{E}_w = E_w[(dx/dt) - (dx'/dt)] \\ &= E_w(v - r) = \rho[(v - r)^3]/2 \end{aligned} \quad (2)$$

where the dot notation denotes a steady flow rather than a time derivative of E_w . Equation (2) gives the power of the wind relative to the speed of the fire per unit vertical area of the atmosphere at height z above the fire.

Byram's derivation of P_w involved the atmospheric windspeed v , but he used the relative windspeed ($v - r$) in Davis (1959). The author considers the formulation using $(v - r)$ more accurate for fast-moving

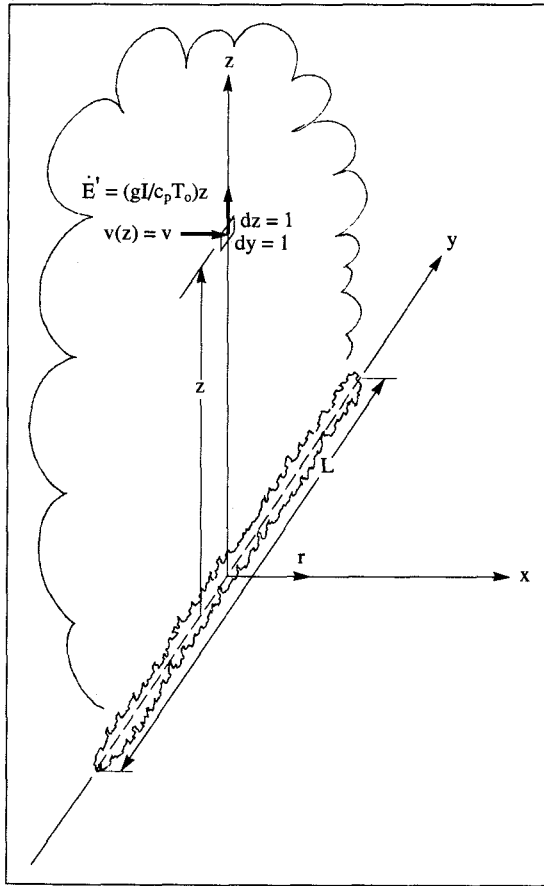


Figure 1. Segment of length L from an infinitely long fire heat source with rate of spread r in the x direction. At height Z , heated air rising from the source interacts with the windfield $V(Z)$. Speed of the horizontal wind is v at elevation Z above the fire. The vertical flow rate of kinetic energy per unit length of source [\dot{E}' in equation (27)] increases linearly with Z . Power of the wind Pw and power of the fire Pf are related to unit vertical area $dydz$ normal to the direction of spread.

wildfires because $v^3(1-r/v)^3$ can be significantly smaller than v^3 when r is but a small fraction of v .

Energy Rate of the Fire P_f

Suppose the fire shown in Figure 1 is burning steadily with an average fireline intensity I (Byram 1959). The atmosphere is assumed to be neutrally stable (or adiabatic) and heated air and combustion products are assumed to ascend adiabatically in the convection column. Further, it is assumed for convenience that there is no entrainment into the ascending fluid. Entrainment, as used here, is the turbulent mixing

of environmental air with convection column gases by the combined processes of diffusion and advection.

Consider a mass m of combustion gases (hereafter referred to as air) issuing from length L of the infinitely long fireline in Figure 1. Archimedes' principle may be used to write the element of energy released (or the amount of work done by the heated mass of air) in ascending a distance dz as

$$dE = g(\rho - \rho_w)V_w dz \quad (3)$$

where ρ_w and V_w are the mass density and volume of the mass m of heated air and E has units of energy. If the pressure p of air is the same inside and outside the convection column at height Z above the earth's surface, then the ideal gas law gives

$$\begin{aligned} \rho - \rho_w &= (p/R_a)[(1/T) - (1/T_w)] \\ &= (p/R_a)[(T_w - T)/T_w T] \end{aligned} \quad (4)$$

where R_a is the gas constant for air, T the absolute temperature of the environmental air at height Z , and T_w the absolute temperature of air in the convection column at Z . From the gas law,

$$V_w = mR_a T_w / p. \quad (5)$$

The hydrostatic approximation for the atmosphere outside the convection column is

$$dz = -dp/\rho g = -(R_a T/g)d(\log p). \quad (6)$$

Substitution of equations (4), (5), and (6) into equation (3) leads to

$$dE = -mR_a(T_w - T)d(\log p). \quad (7)$$

Energy E is properly interpreted as the energy of the fire converted to kinetic energy between the earth's surface ($Z = 0$) where $E = 0$ and height Z . At the top of the neutrally stable atmosphere, all thermal energy from the fire has been converted to kinetic energy and E has its maximum value.

It is convenient to introduce potential temperature θ into equation (7). The potential temperature is the temperature a parcel of air will attain when brought adiabatically from some height in the atmosphere where the pressure is p to the earth's surface (or to sea

level) where the pressure is p_o . This reference pressure is often taken as 1,000 mb. Potential temperature θ is related to environmental temperature T by the equation

$$T = \theta(p/p_o)^\kappa \quad (8)$$

where κ is the ratio of the gas constant for air to the specific heat at constant pressure, R_a/c_p . For air, $\kappa = 0.286$. A similar equation for the heated air is

$$T_w = \theta_w(p/p_o)^\kappa \quad (9)$$

so $(T_w - T)$ may be written as

$$T_w - T = (\theta_w - \theta)(p/p_o)^\kappa. \quad (10)$$

Elimination of $(T_w - T)$ between equations (7) and (10) gives

$$dE = -mR_a(\theta_w - \theta)(p/p_o)^\kappa d(\log p) \quad (11)$$

which also may be written as

$$dE = -mR_a(\theta_w - \theta) \left[p^{\kappa-1}/p_o^\kappa \right] dp. \quad (12)$$

If dp is expressed in terms of $d(p/p_o)^\kappa$ and then substituted into equation (12), dE becomes

$$dE = -m(R_a/\kappa)(\theta_w - \theta)d(p/p_o)^\kappa. \quad (13)$$

The potential temperature θ_w is constant in an adiabatic lifting process and θ is constant in an adiabatic atmosphere, so the difference $(\theta_w - \theta)$ also is constant. In this case, equation (13) can be integrated to give

$$E = -m(R_a/\kappa)(\theta_w - \theta)(p/p_o)^\kappa + C \quad (14)$$

where C is a constant. Because $E = 0$ when $p = p_o$,

$$E = m(R_a/\kappa)(\theta_w - \theta)[1 - (p/p_o)^\kappa]. \quad (15)$$

If the mass m ascends to the top of the adiabatic atmosphere where $p = 0$, then a maximum amount of energy has been released and $E = E_{max}$, where

$$E_{max} = m(R_a/\kappa)(\theta_w - \theta) = mc_p(\theta_w - \theta) \quad (16)$$

owing to $R_a/\kappa = c_p$. Equation (15) now can be written as

$$E = E_{max}[1 - (p/p_o)^\kappa]. \quad (17)$$

The next step in the derivation is to establish a relationship between E and z using the hydrostatic approximation and the equation of state for an ideal gas undergoing an adiabatic process. Because this equation operates throughout the adiabatic atmosphere,

$$pV^{1/(1-\kappa)} = p_oV_o^{1/(1-\kappa)} \quad (18)$$

which may be rewritten using $m = \rho V = \rho_o V_o$ as

$$\rho = \rho_o(p/p_o)^{1-\kappa} \quad (19)$$

where V_o is the volume of environmental air of mass m at the earth's surface. Substituting equation (19) into equation (6) and integrating leads to

$$\int_{p_o}^p p^{\kappa-1} dp = - \left[\rho_o g / p_o^{1-\kappa} \right] \int_0^z dz \quad (20)$$

which becomes

$$p_o V_o [1 - (p/p_o)^\kappa] / \kappa = mgz \quad (21)$$

where ρ_o is replaced by m/V_o . Because

$$p_o V_o = mR_a T_o = mc_p \kappa T_o, \quad (22)$$

equation (21) may be written as

$$c_p T_o [1 - (p/p_o)^\kappa] = gz. \quad (23)$$

When $p = 0$, $z = z_{max}$ where z_{max} is the depth of the adiabatic atmosphere (about 30.4 km for $T_o = 298$ K). Thus, z_{max} is given by

$$z_{max} = c_p T_o / g \quad (24)$$

and equation (23) becomes

$$z/z_{max} = 1 - (p/p_o)^\kappa. \quad (25)$$

Combination of equations (17), (24), and (25) leads to

$$E = E_{max}(z/z_{max}) = E_{max}(gz/c_p T_o). \quad (26)$$

It is noted that there exist alternative, simpler ways to derive equation (26). For example, equation (23) may be substituted directly into equation (17). The procedure used here closely follows the Byram derivation.

Equation (26) also holds if the released energies E and E_{max} are replaced by rates of energy flow \dot{E} and \dot{E}_{max} . Furthermore, these energy flow rates may be interpreted as rates per unit length of source because mass m emanates from length L of fireline. If I is the rate at which the line source gives up heat to the atmosphere per unit length of source, then $\dot{E}_{max}/L = I$ and

$$\dot{E}/L = \dot{E}' = gIz/c_p T_o. \quad (27)$$

This vertical flow of kinetic energy at height z is shown in Figure 1.

Differentiating with respect to z gives

$$P_f = \partial \dot{E}' / \partial z = gI/c_p T_o \quad (28)$$

and P_f represents the rate at which buoyant air, heated by each unit length of source, does work in ascending unit vertical distance in the convection column.

A possible misconception concerning P_f has been that it represents the rate of upward flow of energy through unit horizontal cross section of the convection column. As equation (28) shows, this is not the case. Thus, P_f is the rate at which thermal energy from mass m is converted to kinetic energy in the convection column as it ascends unit vertical distance in the column. Alternatively, P_f may be regarded as the power of the fire (in terms of the rate at which it does work) scaled to the atmospheric vertical area in Figure 1, $dydz$. Because the atmosphere is neutrally stable, I is constant with height and P_f also is independent of height.

Convection Number N_c

By definition, convection number N_c is

$$N_c = (\partial \dot{E}' / \partial z) / \dot{E}_w = P_f / P_w. \quad (29)$$

Use of equations (2) and (28) in equation (29) gives

$$N_c = 2gI/\rho c_p T_o (v-r)^3 \quad (30)$$

in accordance with the P_f/P_w ratio discussed by Byram (1959).

The reader may note that this result for N_c differs by the factor "2" from the equation for N_c described by Byram et al. (1964). The latter equation was derived by the more general method of dimensional analysis in which constants are often undetermined. Equation (30) is the correct expression for N_c .

Effects of Entrainment and Atmospheric Stability

To simplify the derivation of P_f it was assumed that environmental air did not mix with heated air from the line source. It can be shown that for an adiabatic atmosphere the buoyancy of a given mass of heated air is not changed by mixing or entrainment.¹ The buoyancy is independent of pressure p , and hence independent of height z . At the earth's surface where the pressure is p_o , let V_o, ρ_o , and T_o be the volume, density, and absolute temperature of a parcel of warm air of mass m ; let ρ'_o and T'_o be the density and temperature of the environmental air. Similarly, let V, ρ , and T , and ρ' and T' be the same properties at height z where the pressure is p . The buoyancies B_o and B at the surface and at z are

$$B_o = V_o(\rho'_o - \rho_o)g \quad (31)$$

$$B = V(\rho' - \rho)g.$$

Because $m = \rho_o V_o = \rho V, \rho_o T_o = \rho_o T'_o$, and $\rho T = \rho' T'$, the buoyancies may be written as

$$B_o = m[(\rho'_o/\rho_o) - 1]g = m[(T_o/T'_o) - 1]g \quad (32)$$

$$B = m[(\rho'/\rho) - 1]g = m[(T/T') - 1]g.$$

From the definition of potential temperature in equation (8), it follows that

$$T_o/T'_o = T/T' \quad (33)$$

¹ In fact, the buoyancy of a given mass of air plus its entrained mass is independent of mixing and entrainment.

and therefore $B_o = B$. Thus, P_f and N_c are independent of entrainment provided the atmosphere is neutrally stable.

The foregoing analysis is considered by the author to apply only to a nonentraining convection column. It is difficult to know what Byram's thoughts were concerning the analysis, but his use of m for mass of the heated air parcel at the earth's surface and at height z is confusing because entrainment is not explicitly considered. An analysis showing that buoyancy of the total mass (the initial plus entrained mass) is independent of height z is presented in the appendix.

It is not necessary to restrict N_c to an adiabatic atmosphere. For large fires, the analysis for the adiabatic atmosphere will be a good approximation even though the atmosphere is not neutrally stable and entrainment occurs. If fireline intensity I is regarded as a function of z , then N_c at height z should have the same significance regardless of the stratification of the atmosphere. For the adiabatic case, I is independent of z . For all atmospheric conditions, I approaches the convective rate of heat output per unit length of source as z approaches zero. For unstable conditions, I will increase slightly with height. This means that a fire burning on a warm sunny afternoon can capture energy stored in an unstable atmosphere. For high intensity fires, however, the energy so captured is small compared to that supplied by the burning fuel. This would not be the case for weakly-burning fires. If the atmosphere is stably stratified, I decreases as height z increases. Thus, the fire gives up energy to the stable atmosphere. Depending on the initial value of I and the vertical temperature distribution of the atmosphere, there will be a height for which I equals zero. At this height buoyancy becomes zero and the convection column tends to flatten out. Momentum of the convection column flow, however, causes the air to rise to a slightly higher level than that of zero buoyancy.

Discussion and Conclusions

Several assumptions have been made that were not pointed out by Byram (1959) in his chapter on fire behavior or in his derivation of N_c . Some of these are briefly discussed below:

1. Conceptually, the fire heat source shown in Figure 1 is a straight line of zero width. In practice, the heat source consists of a fire burning along the earth's surface or in the canopy of standing trees with one or more heads. The fireline could be discontinuous in places. Nevertheless, the straight line assumption is useful provided the extent of frontal depth in the direction of spread is many times smaller than the length of total fireline.

2. In the derivation of P_f the adiabatic atmosphere assumption is applied at the earth's surface, $z = 0$. Clearly, such an approximation is not valid near the fire due to relatively large temperatures in and just above the flame zone. Entrainment of cooler air and heat loss by radiation will occur in this region. Temperature of the combustion gases decreases rapidly as elevation increases, however, so at some distance above the fire (say 5 to 10 flame heights), the assumption of an adiabatic ascent is approximately correct if the atmosphere is neutrally stable.

3. The effects of condensation/evaporation processes have been ignored in the derivation. When the atmosphere or convection column becomes saturated with moisture due to high relative humidities and/or moisture released during combustion, the ascent of heated air can no longer be considered adiabatic and the derivation is rendered inapplicable. This can occur at or beneath the lifting condensation level which may be found between 1.5 and 3 km on summer days in the western United States. This level is usually lower in more moist climates.

4. In the convection column, buoyancy will be reduced if the average molecular weight of the column gases exceeds that of the environmental air. Use of the gas constant and constant-pressure specific heat for air throughout the analysis implies that the differences in molecular weight are negligible.

Byram's analysis of how the value of N_c is affected by atmospheric stability and entrainment deserves comment. Byram claims that buoyancy is independent of height z and, therefore, that N_c will be independent of mixing and entrainment. The first part of this statement is valid, as is shown in the appendix. It is doubtful, however, that consideration of buoyancy alone is sufficient to explain the effect of entrainment on N_c . Accretion of horizontally moving air in the convection column, for instance, will drastically reduce the vertical component of velocity in the column, thereby changing the flux of kinetic energy. The value of P_f will be smaller, causing a substantial reduction in the value of N_c . Other factors such as column tilt angle and fluxes of mass, momentum, and total energy also will be altered. Byram's discussion of stability effects in terms of fireline intensity I as a function of height z seems reasonable. The effects of an unstable or stable atmosphere on P_f however, may be effects on a P_f value already reduced by entrainment rather than on a P_f calculated using equation (28). The latter value is applicable only at the earth's surface. It is further noted that intensity I is constant for a steadily burning fire and is determined by the fuel consumption rate at or near the ground. In this sense, it is misleading to refer to I as a function of z . Variability in P_f hence

in N_c , is due to variation in the flux of kinetic energy rather than in fireline intensity. Another obvious source of variability in N_c is change in P_w as z increases.

Despite these and perhaps other limitations in the derivation of N_c , Byram (1959) identified a correlation between N_c and certain convection column characteristics related to erratic, and often unexpected, fire behavior. For the wildfires he has examined thus far, Rothermel (1991b) successfully related convection column appearance (primarily angle of tilt from the vertical) to values of N_c . An immediate inference is that more research on N_c and its role in wildfire behavior should prove exceedingly fruitful.

This is not to say, however, that N_c is the key to understanding the behavior of large fires. Other atmospheric processes that can affect fire behavior are frontal passages (Brotak 1977), large-scale subsidence (Krumm 1959), presence of the jet stream above the fire (Schaefer 1957), and thunderstorm downbursts (Haines 1988). The convection column above a large fire, however, is a local phenomenon compared to the scale of these processes and, in a sense, the local and large-scale phenomena are related in that the latter determine conditions affecting the former. All the physical and chemical processes influencing large wildfire behavior may never be identified or completely understood, but there exists a base of information upon which to build. In this age of sophisticated atmospheric measurements and techniques of computational cloud physics, it seems that much useful knowledge could be gained from a well-organized program of large fire observation and analysis.

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Appendix

This appendix demonstrates that the buoyancy of a given mass of heated air plus its entrained air is independent of height z in the convection column, provided the atmosphere is adiabatic. Byram's analy-

sis in the text is a convenient starting point. Convection column and atmospheric variables are defined as in the analysis. The buoyancy B_o at the earth's surface and the buoyancy B at height z in the column are given by equations (31). Suppose a heated parcel of mass m at the surface has entrained a mass equal to αm at height z , where the mass of entrained air is some multiple of the initial mass. The volumes V_o and V may be written as

$$V_o = m/\rho_o \quad (\text{A - 1})$$

$$V = (1 + \alpha)m/\rho.$$

Equations (31) become

$$\begin{aligned} B_o &= m[(\rho_o'/\rho_o) - 1]g = m[(T_o/T_o') - 1]g \\ &= m[(\theta_o/\theta_o') - 1]g \end{aligned} \quad (\text{A - 2})$$

$$\begin{aligned} B &= (1 + \alpha)m[(\rho'/\rho) - 1]g = (1 + \alpha)m[(T/T') - 1]g \\ &= (1 + \alpha)m[(\theta/\theta') - 1]g \end{aligned}$$

where pressure in the column at height z is assumed equal to the atmospheric pressure at z and the θ symbols denote potential temperatures. Because the atmosphere is adiabatic, the definition of potential temperature may be used to show that

$$\theta_o' = \theta'. \quad (\text{A - 3})$$

In the convection column, however, the potential temperature achieved at z will be a mass-weighted average of the column potential temperature at the surface and the potential temperature of the environment at z . Thus,

$$(1 + \alpha)m\theta = m\theta_o + \alpha m\theta'. \quad (\text{A - 4})$$

In view of equations (A-3) and (A-4),

$$\theta = (\theta_o + \alpha\theta')/(1 + \alpha) = (\theta_o + \alpha\theta_o')/(1 + \alpha). \quad (\text{A - 5})$$

Substitution of equations (A-3) and (A-5) into the second of equations (A-2) gives

$$B = m[(\theta_o/\theta_o') - 1]g = B_o. \quad (\text{A - 6})$$

This equation shows that the total buoyancy is independent of entrainment and, therefore, independent of height z .